

Chord Grammar 3.0 (Halfspan)

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1 Lexicon (Without Semantics)

T	X(m)	:=	I^T
T_III	Xm	:=	$\flat V I^T$
T_bVI	X	:=	III^T
D	X(m) ⁷	:=	$I^D /_c IV^D ^T$
D_Bd	X(m) ⁷	:=	$V I^D /_c II^D ^T$
D_Tt	X(m) ⁷	:=	$\sharp IV^D /_c V II^D ^T$
D_Btk	X(m) ⁷	:=	$\flat III^D /_c \flat V I^D ^T$
S	X(m)	:=	$I^S /_c V^S ^T$
S_bIII	X	:=	$V I^S /_c III^S ^T$
Dim_bVII	X _o	:=	$IV^D /_c \flat V II^D ^T$
Dim_V	X _o	:=	$II^D /_c V^D ^T$
Dim_III	X _o	:=	$V II^D /_c III^D ^T$
Dim_bII	X _o	:=	$\flat V I^D /_c \flat II^D ^T$
Pass_I	X _o	:=	I^T / I^T
	X _o	:=	I^D / I^D
Pass_VI	X _o	:=	$V I^T / V I^T$
	X _o	:=	$V I^D / V I^D$
Pass_bV	X _o	:=	$\flat V^T / \flat V^T$
	X _o	:=	$\flat V^D / \flat V^D$
Pass_bIII	X _o	:=	$\flat III^T / \flat III^T$
	X _o	:=	$\flat III^D / \flat III^D$
Aug_bII	X ⁷	:=	$\flat V I^D /_c \flat II^D ^T$
Aug_VI	X ⁷	:=	$III^D /_c V I^D ^T$
TC_IV	X(m)	:=	V^T / V^T
TC_IVb	X(m)	:=	$V^T \setminus V^T$
TC_II	X(m)	:=	$\flat V II^T / \flat V II^T$
TC_IIb	X(m)	:=	$\flat V II^T \setminus \flat V II^T$

1.1 Minorness

There are no longer any major/minor features on the syntactic categories. They were in effect not being used for anything in the previous version of the lexicon and the categories are much more readable without them.

1.2 Repetition Categories

The previous lexicon had a lot of categories for repeated chords. The basic idea was to have a I/I category and something similar for dominant chord, but every substitution had to have a corresponding version of the repetition category.

Instead, we can just generate these categories automatically by expanding the lexicon using a unary rule, rather like adding type-raised categories to a lexicon.

To generate the tonic repetition categories, we apply:

$$X \Rightarrow_{Rep} X/X : \lambda x.x$$

where X ranges over half-categories with function T .

For the dominant and subdominant repetition categories, we apply:

$$X/Y \Rightarrow_{Crep} X/X : \lambda x.x$$

where X ranges over half-categories with function S or D .

1.3 Modalities

I've reverted to my old simpler modality system, where the only modalities are 'c' and '.' (or nothing). This is sufficient to mark slash categories that represent a cadence, so that the coordination rule can be restricted to apply only to these.

If we wish to restrict the coordination rule further so that it's not possible to coordinate a plagal with an authentic cadence (which currently we do), we can use the function marker on the result half-category.

2 Lexicon (With Semantics)

The logical forms produced by the grammar are now paths, which are made up of lists of coordinates in the tonal space. The points are expressed as four values: an (x, y) coordinate of the point within an enharmonic block (4x3 block); and an (X, Y) coordinate identifying the block.

A lexical category's semantics may refer to a point in the space with two coordinates. These give the (x, y) values.

leftonto is a function $Path \rightarrow Path$. It operates on a tonal space path and produces a path that is identical, but with a point prepended that is one step to the right of the *start point* of the path (usually the first point on the path, see below).

rightonto works analogously.

T	$X(m) := I^T$: $[(0, 0)]$
T_III	$X_m := bVI^T$: $[(0, 2)]$
T_bVI	$X := III^T$: $[(0, 1)]$
D	$X(m)^7 := I^D/cIV^{D T}$: $\lambda x.leftonto(x)$
D_Bd	$X(m)^7 := VI^D/cII^{D T}$: $\lambda x.leftonto(x)$
D_Tt	$X(m)^7 := \sharp IV^D/cVII^{D T}$: $\lambda x.leftonto(x)$
D_Btk	$X(m)^7 := bIII^D/cbVI^{D T}$: $\lambda x.leftonto(x)$
S	$X(m) := I^S/cV^{S T}$: $\lambda x.rightonto(x)$
S_bIII	$X := VI^S/cIII^{S T}$: $\lambda x.rightonto(x)$
Dim_bVII	$X_o := IV^D/cbVII^{D T}$: $\lambda x.leftonto(x)$
Dim_V	$X_o := II^D/cV^{D T}$: $\lambda x.leftonto(x)$
Dim_III	$X_o := VII^D/cIII^{D T}$: $\lambda x.leftonto(x)$
Dim_bII	$X_o := bVI^D/cbII^{D T}$: $\lambda x.leftonto(x)$
Pass_I	$X_o := I^T/I^T$: $\lambda x.x$
	$X_o := I^D/I^D$: $\lambda x.x$
Pass_VI	$X_o := VI^T/VI^T$: $\lambda x.x$

	X_{\circ}	$:= VI^D / VI^D$	$: \lambda x.x$
Pass_bV	X_{\circ}	$:= \flat V^T / \flat V^T$	$: \lambda x.x$
	X_{\circ}	$:= \flat V^D / \flat V^D$	$: \lambda x.x$
Pass_bIII	X_{\circ}	$:= \flat III^T / \flat III^T$	$: \lambda x.x$
	X_{\circ}	$:= \flat III^D / \flat III^D$	$: \lambda x.x$
Aug_bII	X^7	$:= \flat VI^D / \flat III^D ^T$	$: \lambda x.leftonto(x)$
Aug_VI	X^7	$:= III^D / \flat VI^D ^T$	$: \lambda x.leftonto(x)$
TC_IV	$X(m)$	$:= V^T / V^T$	$: \lambda x.x$
TC_IVb	$X(m)$	$:= V^T \setminus V^T$	$: \lambda x.x$
TC_II	$X(m)$	$:= \flat VII^T / \flat VII^T$	$: \lambda x.x$
TC_IIb	$X(m)$	$:= \flat VII^T \setminus \flat VII^T$	$: \lambda x.x$

3 Rules

A path is primarily made up of a list of points. It also may have a *start point* that is distinct from the points of the path.

In general, the start point of a path is just the first point of the path. A path may have a different start point. This is written using a superscript: $^{(5,3)}[(3,0), (2,0), (1,0)]$. The start point of this path is $(5,3)$.

The function *jump* alters the start point of a path. Its first argument is a coordinate c and second a path p . It produces a path with the points of p whose start point is c .

The following grammatical rules are used:

Application:

$$\begin{aligned} X/Y : f \quad Y-Z : x \Rightarrow_{>} X-Z : f(x) \\ X-Y : x \quad Z \setminus Y : f \Rightarrow_{<} X-Z : f(x) \end{aligned}$$

Composition:

$$\begin{aligned} X/Y : f \quad Y/Z : g \Rightarrow_{>B} X/Z : \lambda x.f(g(x)) \\ X \setminus Y : g \quad Z \setminus X : f \Rightarrow_{<B} Z \setminus Y : \lambda x.f(g(x)) \end{aligned}$$

Coordination:

$$X^f / \flat Y : f \quad Z^f / \flat Y : g \Rightarrow_{\&} X / \flat Y : \lambda x.f(\text{jump}(\text{head}(x), g(x)))$$

Development:

$$V-W : x \quad X-Y : y \Rightarrow_{dev} V-Z : dev(x, y)$$

3.1 Development Semantics

The development rule uses the function *dev* as its semantics. It behaves as follows.

Both arguments are paths. A translated version of the first path x' is produced, retaining the (x, y) values of the coordinates, but altering all of the (X, Y) block locations by the same shift. The shift is chosen such that the final point on x falls in the space immediately surrounding the first point on y . Figure 1 shows the instance of note that would be chosen to be closest to the point $(0, 0)$.

The path returned is the concatenation of the shifted version of x with y .

$\sharp IV^-$	$\sharp I$	$\sharp V$	$\sharp III$	$\sharp VI$	$\sharp III^+$
II^-	VI	III	VII	$\sharp IV$	$\sharp I^+$
$bVII^-$	IV	I	V	II	VI^+
bV^-	bII	bVI	$bIII$	$bVII$	IV^+
$bbIII^-$	$bbVII$	bIV	bI	bV	bII^+

Figure 1: Nearest instance of each equally tempered tone to the point I , $(0,0)$